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ABSTRACT

Reported is a study in the Oakleaf Elementary School to test the hypothesis that variability of achievement within a particular grade approximates the number of years the pupils have been in school (e.g., in the third grade, a spread of three years is expected). Data were collected and analyzed regarding range of achievement prior to instruction under IPI, units mastered on placement tests, units mastered after one year of instruction under IPI, range of I.Q. grades, and range of achievement after two years in the program. The data supported the hypothesis stated above. (FP)

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WORKING PAPER 4

VARIABILITY OF PUPIL ACHIEVEMENT IN MATHEMATICS

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There is considerable evidence to support the assumption that pupils in a given grade achieve at varying levels in the same subjects. One hypothesis for which supporting evidence is available is that the variability in a particular grade at least approximates the number of years the pupils have been in school (e.g., in the third grade one would expect a spread of three years in achievement.)[1,2] These results are generally reported for graded schools in which the materials, textbooks, school structure and pupils are graded.

Prior to the implementation of the Individually Prescribed Instruction program in the Oakleaf Elementary School in September, 1964, the students attended an elementary school which was organized as a graded school. The results of Metropolitan Achievement tests in mathematics administered to these students in May of 1964 are reported in Table 1. This table will give some indication of what the variability was like prior to introducing IPI.

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[1] Foster E. Grosnickle and Leo J. Brueckner, Discovering Meanings in Arithmetic, New York: Holt, Rinehart and Winston, 1959, p. 373.

[2] John I Goodlad and Robert H. Anderson, The Nongraded Elementary School, Revised, New York: Harcourt, Brace and World, Inc., 1963, pp. 6-13.

Although the N's are small, the range of scores does seem to follow the general principle for variability of students in a given subject. However, caution must be taken when attempting to generalize from the information reported here. Any one form of a standardized test is typically developed for use at one or two grade levels. For example, a first grade level test covers content that is appropriate for typical first grade students. Variability of scores on such tests then represents variability in how well pupils have mastered what is largely first grade content. A grade equivalent of 3.1 on a first grade test means that the student is doing as well as the average beginning third grader in his command of first grade content. This type of information in terms of grade norms does not provide information about student knowledge for a continuum of achievement. But, keeping this limitation in mind, it still indicates that there is evidence to support the assumption that any educational endeavor to be maximally effective must provide for this variability of achievement. This is one of the objectives of the IPI project.

Two questions were posed with the introduction of IPI related to variability: (1) If achievement tests are developed to measure student abilities in terms of a continuum of achievement, how much variability exists? And, (2) If instructional conditions are created to adjust to these differences, what is the effect of this instruction on average class attainment and variability? During the planning stages of the project it became evident that some sort of measures of educational achievement, other than standardized tests, must be

developed to measure mastery of content for diagnosing pupil weaknesses and competencies in each of the categories of mathematics. A series of placement tests were developed for this purpose. These instruments were administered to all students in the Oakleaf Elementary School prior to the introduction of the IPI project. Tables 2 and 3 present the results of this testing.

In relation to the first question, these data seem to indicate that for each grade there is a variability of achievement in mathematics and that this variability increases as the number of years in school increases. However, we have not yet established any norms to determine just how many units correspond to a year's work, therefore, it is impossible to check the hypothesis stated earlier concerning variability and number of years in school. Of particular interest in Table 3 is the overlap of students of the various grades. For instance, at least one student in grade three has mastered as many units in mathematics as have students in the fourth and sixth grades. Also, the more advanced pupils in grade four have exceeded the mean achievement of grade five, and, similarly, the more advanced fifth graders have exceeded the mean achievement in terms of units mastered of the sixth graders. This evidence tends to support the need for IPI at the same time giving necessary information about the placement tests themselves.

A second finding from the results of these placement tests was that there was considerable intra-individual variance even within the one subject of mathematics. Of the twelve areas of mathematics measured

by the placement tests it was found that the intra-individual variances increased with the number of years the student was in school. An examination of the information presented in Tables 4 and 5 illustrates this variability.

In reference to the second major question concerning variability and effect of individualized instruction, it was hypothesized that pupils involved in the individualized instruction program would exhibit greater variability than pupils involved in a graded program. To test this hypothesis data concerning pupil variability in achievement prior to and after one year of involvement in the IPI program were collected and analyzed. Tables 6 through 11 give the placement test results in mathematics in terms of units mastered by students in each grade at the beginning of the school year in September, 1965. Table 12 is a summary of the information from Tables 6 through 11. This table reports the average number of units completed and the variability of units completed by grade after one year of IPI. To compare these results with similar students from a graded program these results were compared to the results of placement for the Oakleaf students prior to entering IPI in 1964.

In order to determine whether or not the classes representing the various grades were similar, the IQ's of the students are reported in Table 14 which gives the means and standard deviation of IQ's of all pupils involved in the program from the beginning by grade. An analysis of the means and variances for each grade in 1964 to 1965 indicates no significant difference for the grades for each year. For instance, the IQ of grade one students in 1964 does not differ from the IQ of grade one students in 1965, etc., for each grade. Teacher judgment of

the classes involved confirmed this finding in relation to other variables such as attitude, maturity, etc.

It then seemed appropriate to compare the means and standard deviations of achievement for each grade for the two years. This information is presented in Table 15. F ratios for homogeneity of the two variances by grade for grades two and three are significant beyond the .01 level. This tends to support the hypothesis under study. However, for grades four, five, and six there was less variability after one year of IPI. A more detailed study of the data and the system employed for IPI does lead to some insight as to why this would be. First, it appears that the extreme lower cases present in these groups when receiving conventional instruction have been eliminated with IPI. This would indicate that when the students are given work at their own level of learning they are able to progress satisfactorily through the mathematics curriculum. Secondly, there is an assumption here that on the average the units for the various levels contain approximately the same number of skills and require approximately the same amount of time to complete. At this time, as Dr. Lindvall and Mr. Yeager's report indicates, we do not have rate measures to analyze the time-difficulty problem, but as to the number of skills per unit, an examination of the continuum in mathematics reveals that for those levels studied by the most advanced students in the intermediate grades contain more skills than the units at lower levels. This, however, is only part of the answer. There are many questions related to variability that must be studied more in terms of specific tasks rather than in terms of combinations of these tasks as now measured by our unit measures.

TABLE 1

Range of Achievement in Mathematics by Grade for Oakleaf Students

From Results of Metropolitan Achievement Tests - May, 1964

Grade	Lowest Score	Highest Score	Range
2	1.7	3.2	1.5
3	2.4	4.4	2.0
4	3.2	5.7	2.5
5	4.2	7.8	3.6
6	3.1	11.4	8.3

TABLE 2

Mean and Spread of Scores in Mathematics by Grade for Units Mastered on

Placement Tests -- 1964

Grade	N	$\bar{X}$	S	Range
1	30	5.20	1.42	3-8
2	27	8.07	2.20	3-13
3	31	14.16	1.65	11-18
4	25	24.68	5.54	15-34
5	21	30.23	7.03	21-47
6	23	35.78	7.72	17-47

TABLE 3

Range of Units Mastered in Mathematics Within Grade Levels

1964

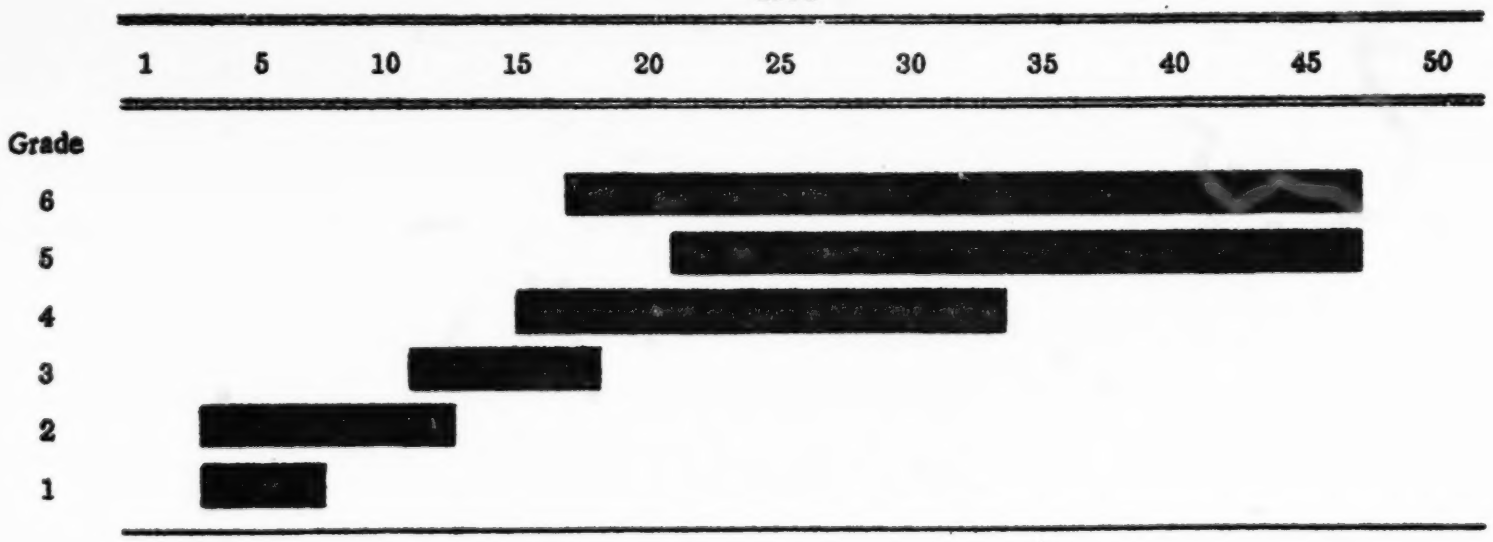

TABLE 4

Placement and Units Mastered in
Mathematics by Grade Three Students

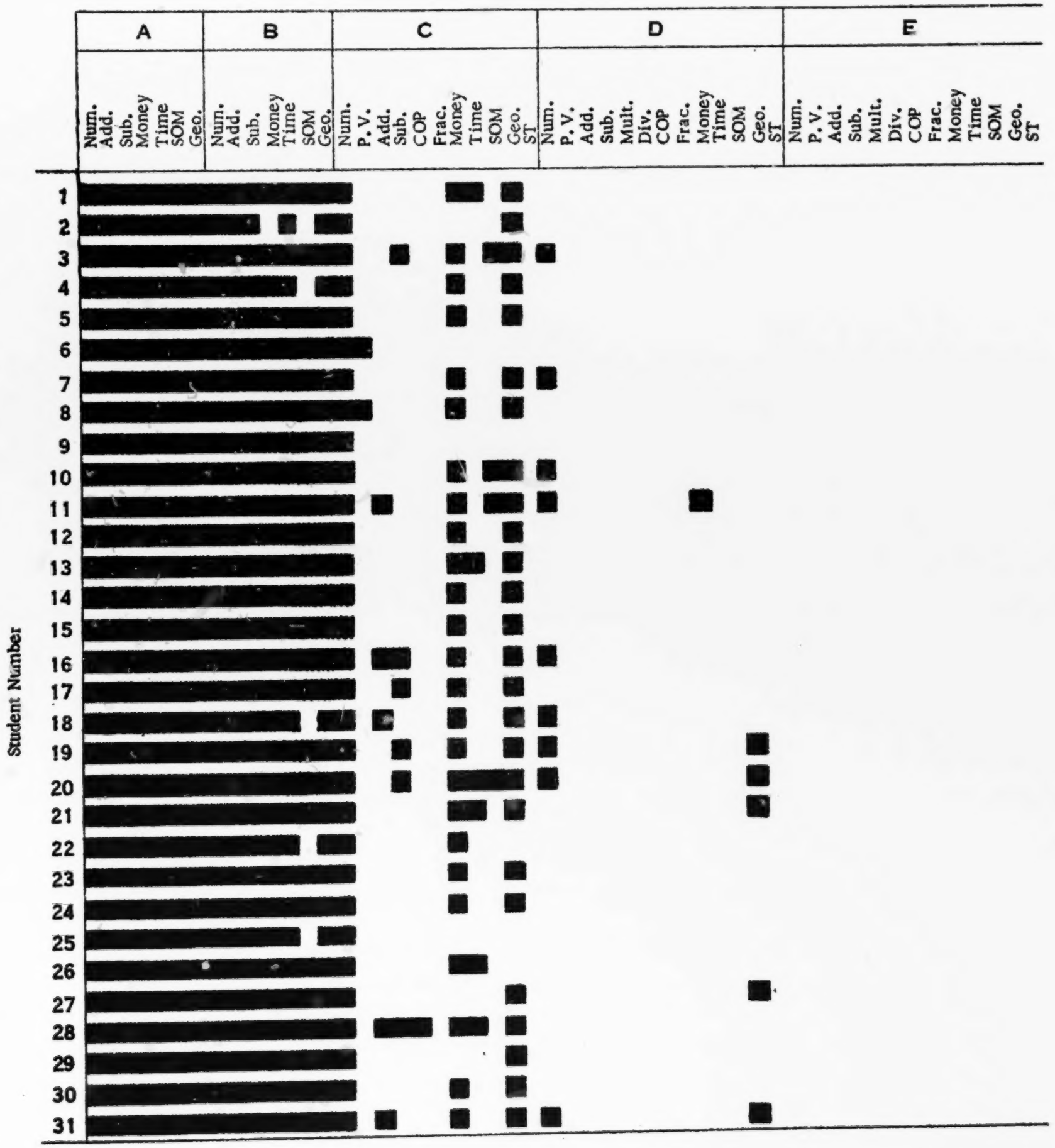

TABLE 5

Placement and Units Mastered in Mathematics
by Grade Four Students

TABLE 6
Placement in Mathematics by
Grade One Students in 1965

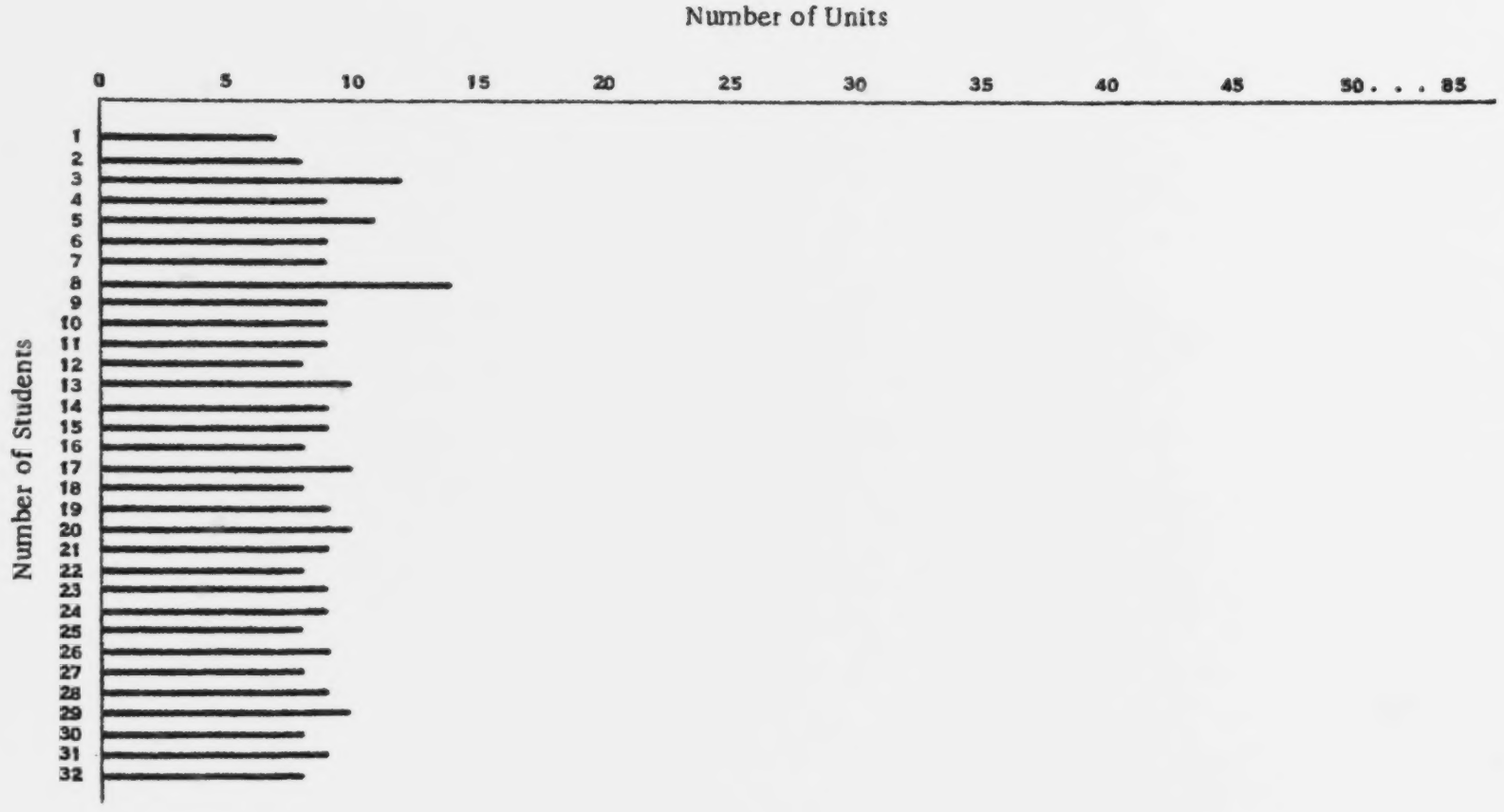

TABLE 7
Placement in Mathematics by
Grade Two Students in 1965

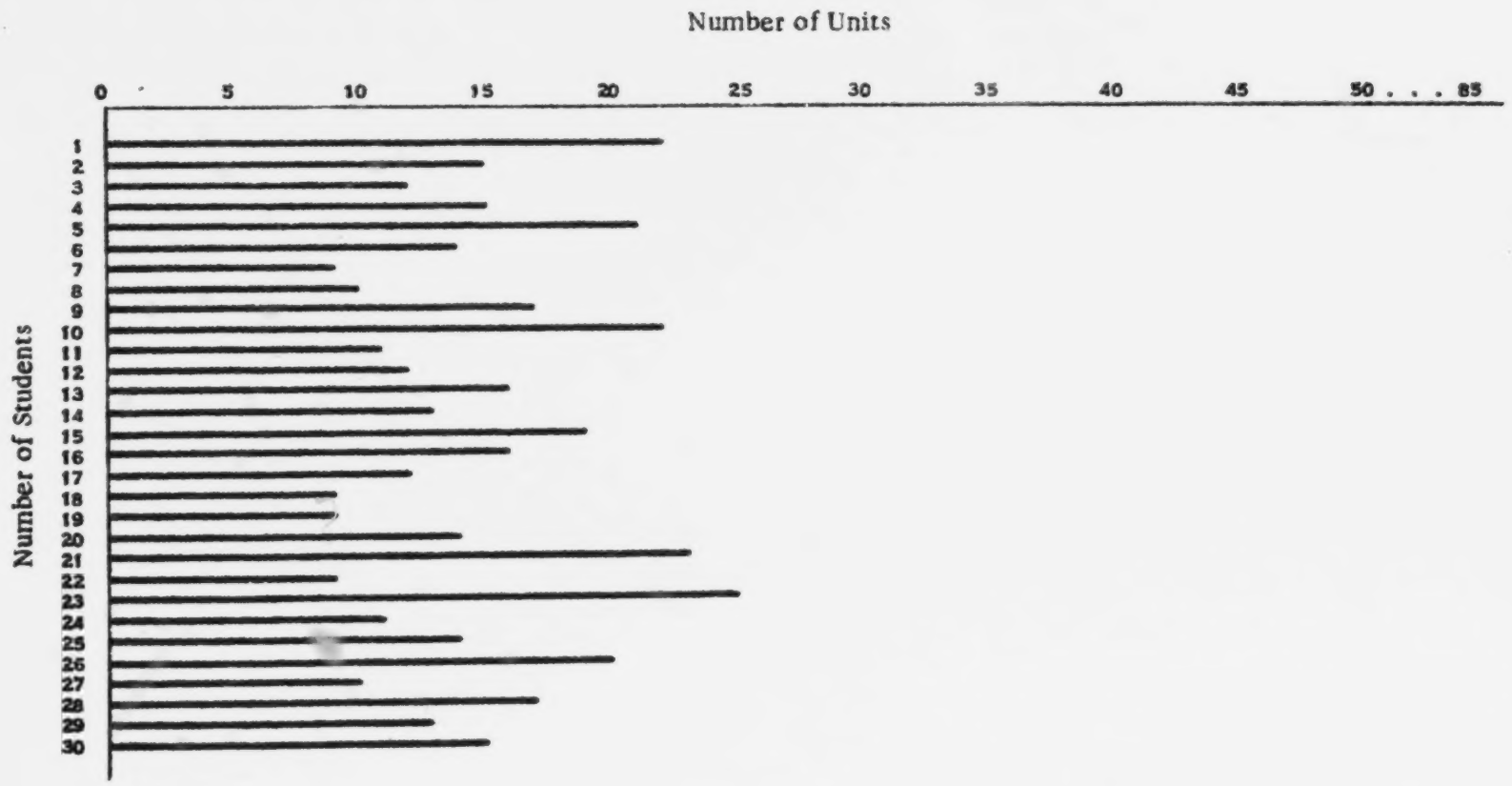

TABLE 8
Placement in Mathematics by
Grade Three Students in 1965

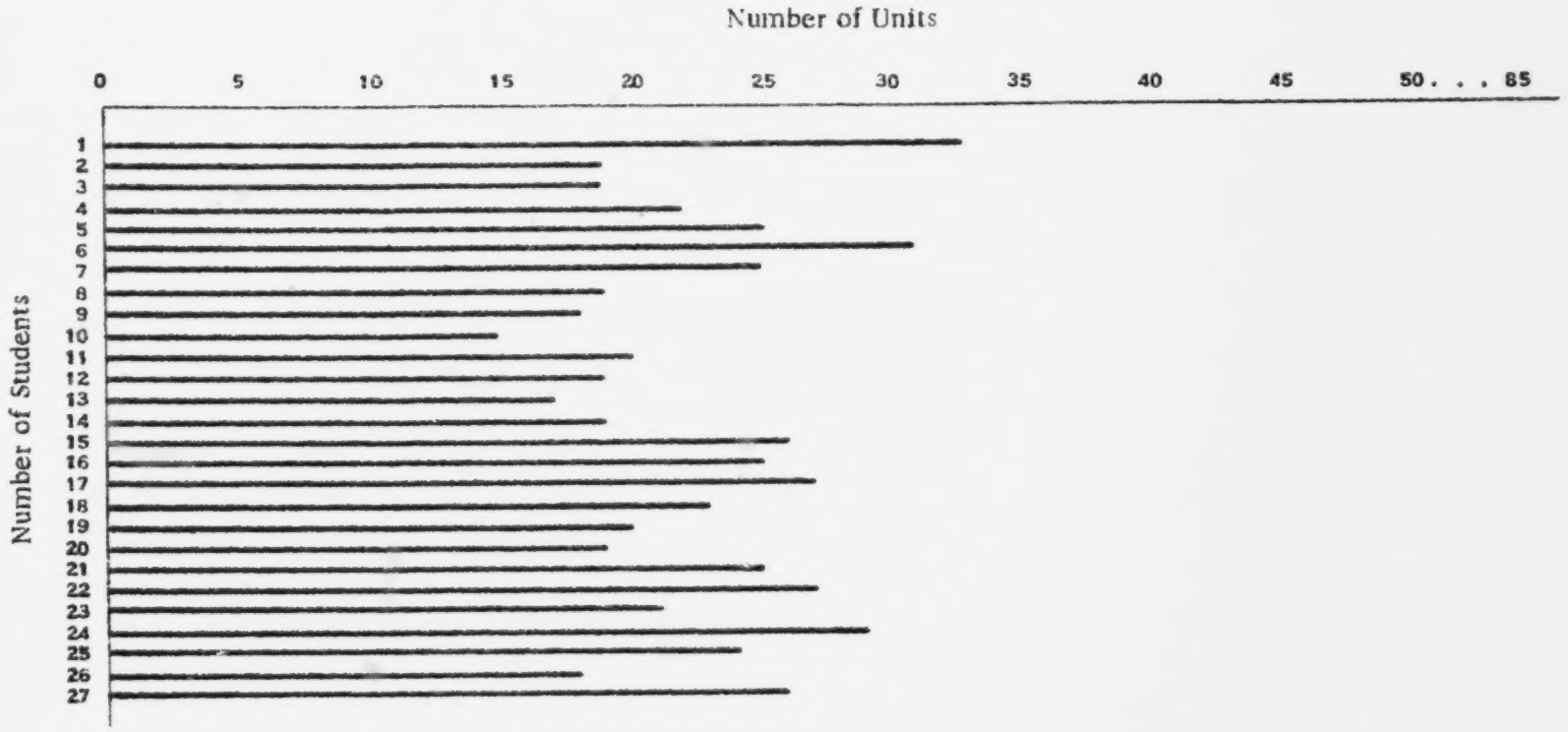

TABLE 9
Placement in Mathematics by
Grade Four Students in 1965

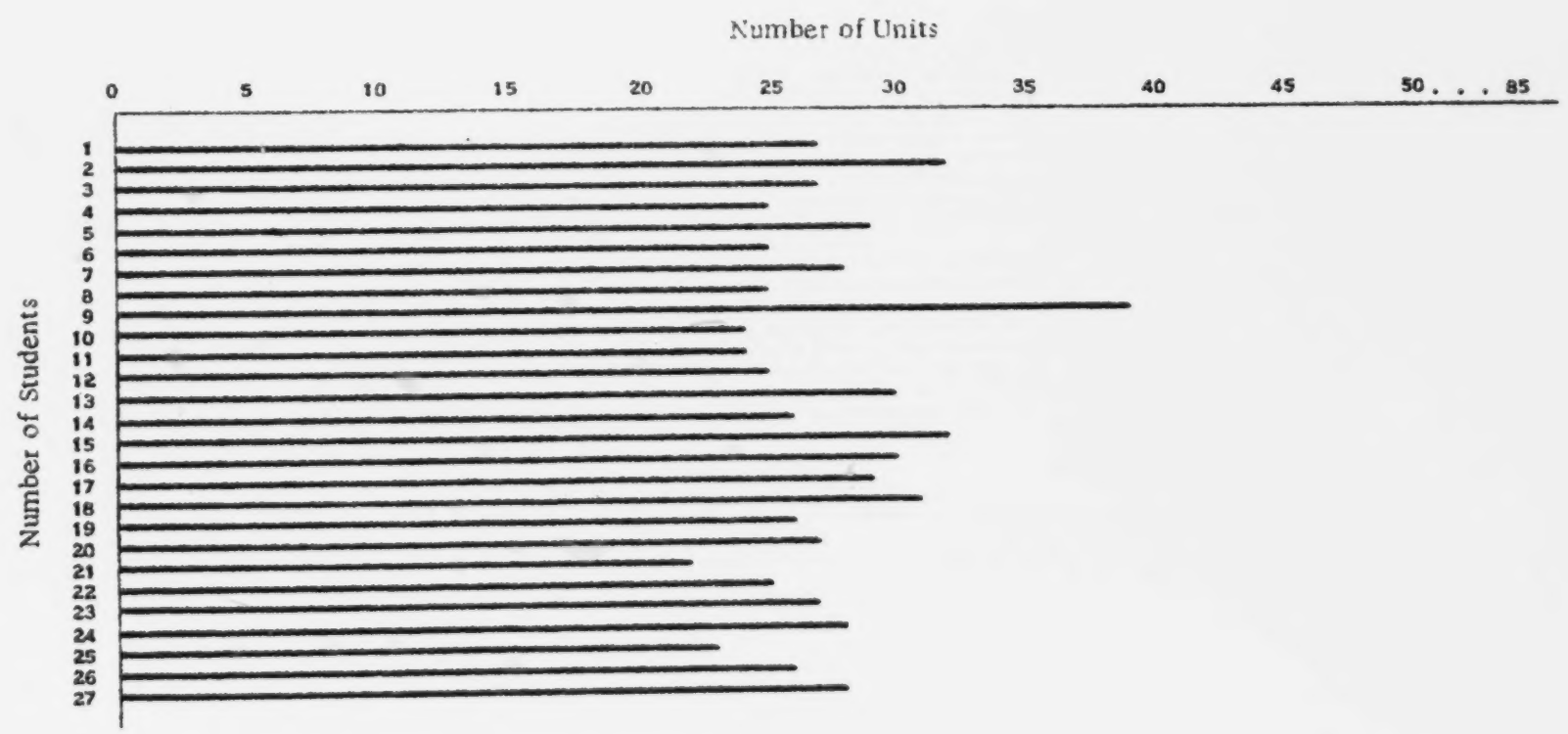

TABLE 10
Placement in Mathematics by
Grade Five Students in 1965

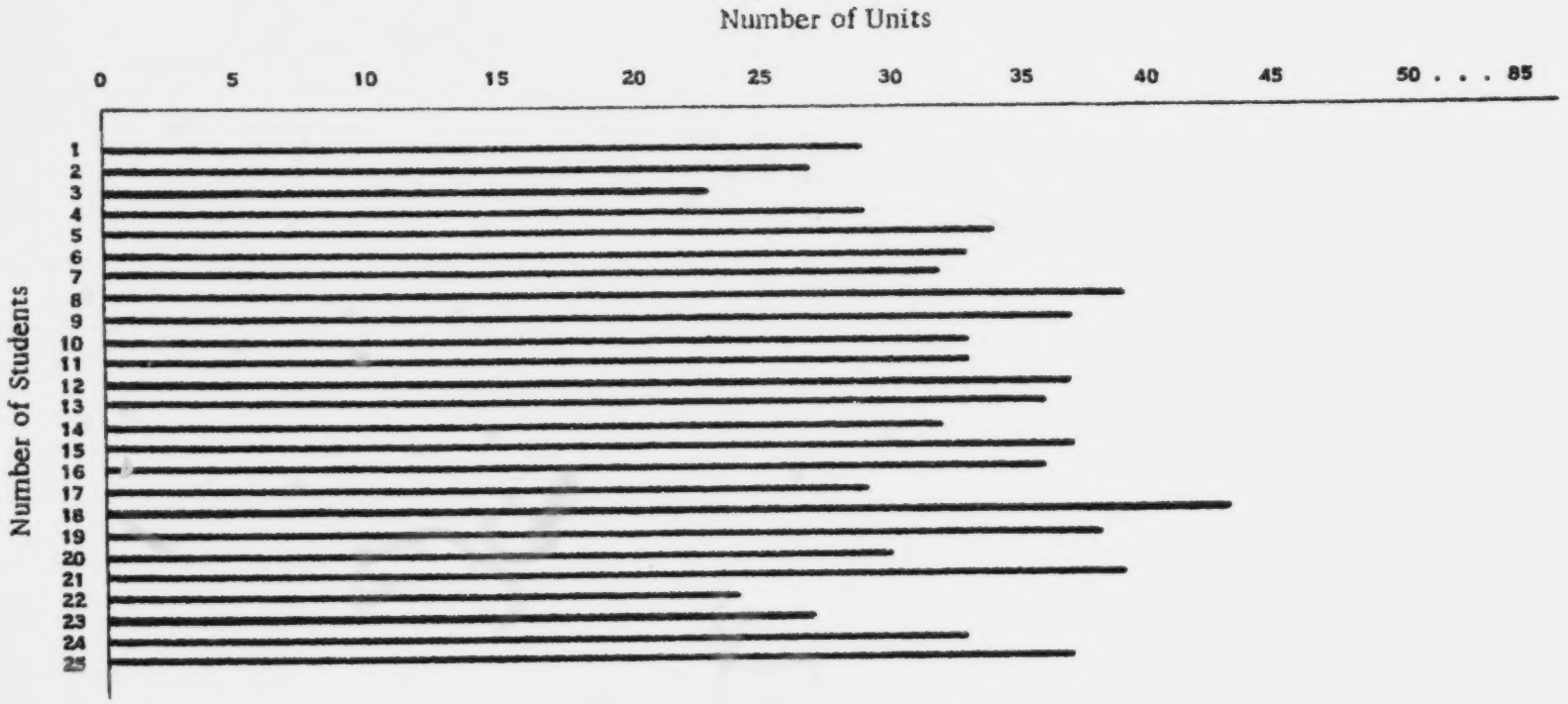

TABLE 11
Placement in Mathematics by
Grade Six Students in 1965

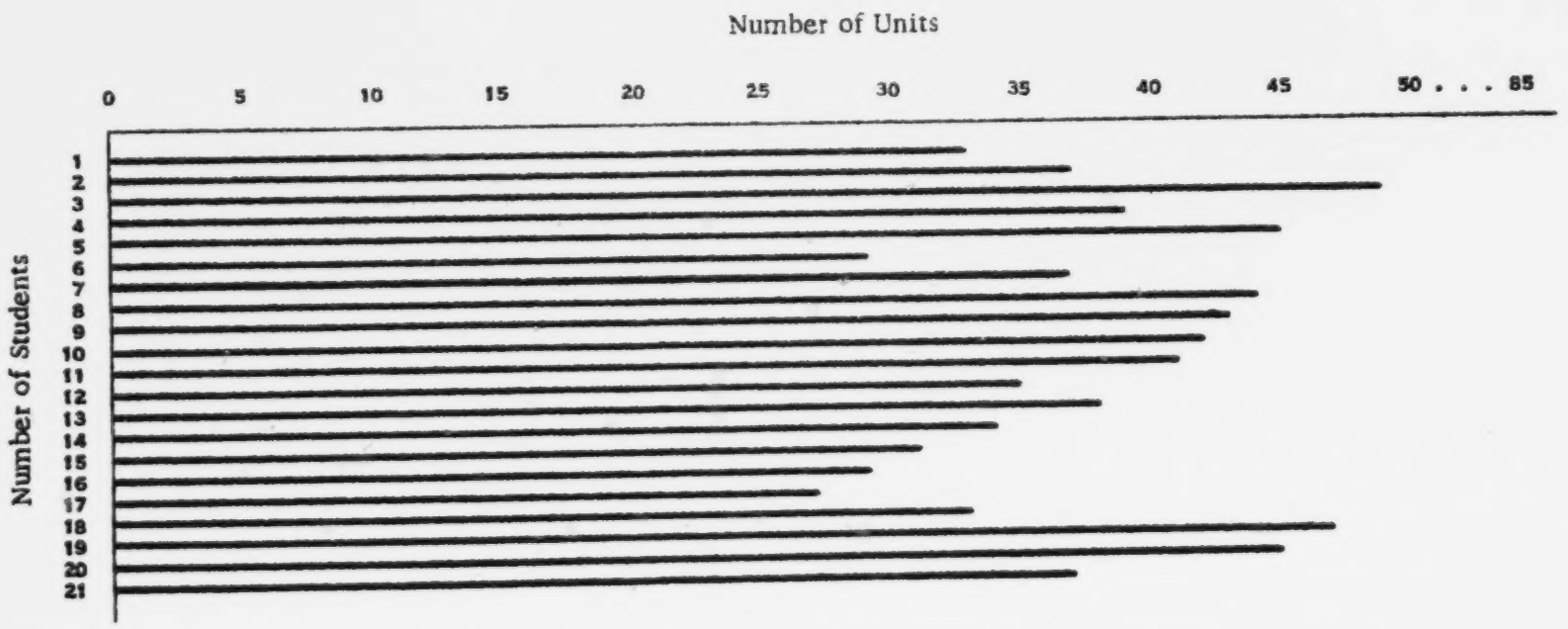

TABLE 12

Mean and Spread of Scores in Mathematics by Grade for Units Mastered on Placement Tests - 1965

Grade	N	$\bar{X}$	SD	Range
1	32	8.15	1.26	7-13
2	30	15.86	3.94	8-24
3	27	21.63	4.52	14-32
4	27	26.40	3.49	21-38
5	25	31.96	4.81	22-42
6	21	36.85	6.31	26-48

TABLE 13

Range of Units Mastered in Mathematics Within Grade Levels

1965

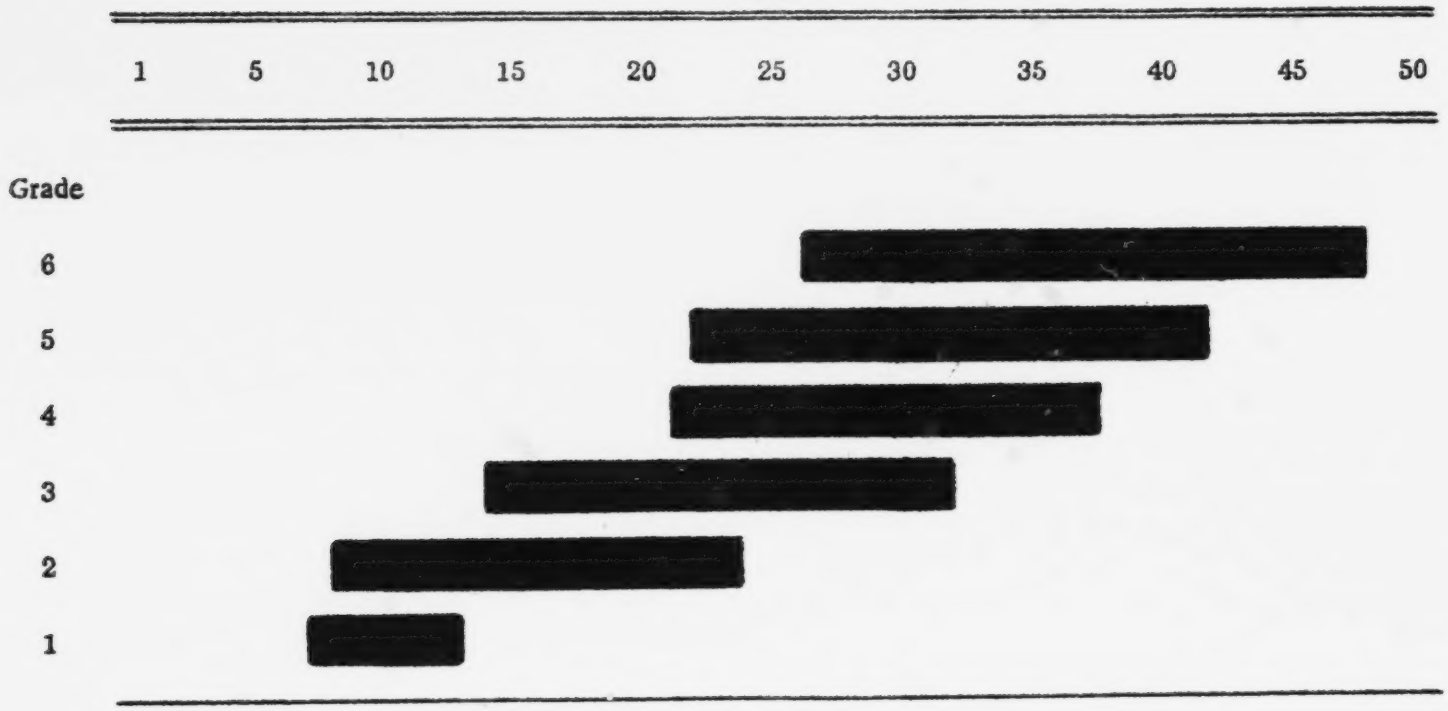

TABLE 14

Mean and Spread of IQ Scores by Grade

Grade 1964	Grade 1965	N	$\bar{X}$	SD	Range
K	1	31	120.54	15.20	83-145
1	2	30	114.46	12.42	79-132
2	3	28	111.11	8.09	94-131
3	4	27	119.96	10.60	98-138
4	5	25	117.12	9.91	92-135
5	6	21	115.85	12.22	84-132
6		23	112.08	16.46	62-132

TABLE 15

Comparison of Means and Standard Deviations of

1964 and 1965 Mathematics Achievement

Grade	1964 $\bar{X}$	1964 SD	1965 $\bar{X}$	1965 SD
1	5.20	1.42	8.15	1.26
2	8.07	2.20	15.86	3.94
3	14.16	1.65	21.63	4.52
4	24.68	5.54	26.40	3.49
5	30.23	7.03	31.96	4.81
6	35.78	7.72	36.85	6.31